

$k = 3$ (called "crowds" in England) but Borho's analytical work may assist in settling this.

It is stated here that no amicable pair is known that does not terminate an aliquot chain. A priori, the reviewer sees no compelling reason to doubt the existence of one since, analogously, the perfect number 28 does not terminate a chain.

D. S.

1. ELVIN J. LEE, *The Discovery of Amicable Numbers*, *Math. Comp.*, v. 24, 1970, pp. 493–494, RMT 40.

37[9].—RUDOLF ONDREJKA, *Mersenne Primes and Perfect Numbers*, ms. of 91 computer sheets (undated) deposited in the UMT file.

Herein are listed in decimal, octal, and binary form, respectively, the exact values of the first 23 Mersenne primes and the corresponding perfect numbers. Also presented are such relevant statistics as the number of decimal digits in each number, the corresponding digital sum, the frequency distribution of these digits and the associated cumulative frequency distribution.

The author includes explicit expressions of the perfect numbers as sums of cubes of successive odd numbers, sums of successive powers of 2, and sums of arithmetic progressions.

Appropriate entries in these tables were compared by the author with corresponding results of Uhler [1]. Also, the eighteenth Mersenne prime was checked against the value of Riesel [2], and the last three Mersenne primes listed here were checked against the corresponding results of Gillies [3].

Furthermore, this reviewer has successfully compared the statistics herein with corresponding data found by Lal [4].

It seems appropriate to note here that an additional Mersenne prime has been recently announced by Tuckerman [5].

J. W. W.

1. H. S. UHLER, "Full values of the first seventeen perfect numbers," *Scripta Math.*, v. 20, 1954, p. 240, where references to Professor Uhler's previous related calculations are given.

2. H. RIESEL, "A new Mersenne prime," *MTAC*, v. 12, 1958, p. 60.

3. D. B. GILLIES, *Three New Mersenne Primes and a Conjecture*, Report No. 138, Digital Computer Laboratory, University of Illinois, Urbana, Illinois, 1964.

4. M. LAL, *Decimal Expansion of Mersenne Primes*, Department of Mathematics, Memorial University of Newfoundland, St. John's, Newfoundland, 1967. (See *Math. Comp.*, v. 22, 1968, p. 232, RMT 20.)

5. B. TUCKERMAN, "The 24th Mersenne prime," *Proc. Nat. Acad. Sci. U.S.A.*, v. 68, 1971, pp. 2319–2320.

38[9].—G. AARON PAXSON, *Table of Aliquot Sequences*, Standard Oil Co. of California, 225 Bush Street, San Francisco, California 94120, computer output, 134 sheets filed in stiff covers and deposited in the UMT file in 1966.

Let $s(n)$ be the sum of the aliquot parts of n , i.e. divisors of n other than n itself. According as $s(n) = n$, $< n$ or $> n$, n is perfect, deficient or abundant. Define $s^0(n) = n$, $s^{k+1}(n) = s(s^k(n))$, $k \geq 0$. The author tabulates $s^k(n)$ for $k = 0, 1, 2, \dots$, and each

abundant n through $n = 3040$. Each sequence is pursued until either (i) a prime p occurs, whereafter $s(p) = 1$, or (ii) a member of an earlier sequence occurs, or (iii) the next term would exceed 10^{10} . No instructions or explanation are given; here are six consecutive entries from the table, with an interpretation:

552	9	43	6526189068	2	1	2	1	2	3	7	31	358031
558			558	1	2	1		2	3	31		
558	8	1	690					462	1			
560			560	4	1	1		2	5	7		
560		1	928	5	1			2	29			
560		2	962	1	1	1		2	13	37		

Column 1 contains n , column 3 contains k (unless it is zero) and the 9 (resp. 8) in column 2 is a flag indicating that case (iii) (resp. (i) or (ii)) has occurred. The last columns contain the distinct prime factors of $s^k(n)$, and the preceding ones give the exponents to which these primes occur (except that the last exponent is missing when flag 9 appears, and there is no factorization when flag 8 appears). These entries may be written, with the factorizations in an obvious notation, as

$$\begin{aligned}
 s^{43}(552) &= 6526189068 = 2(2)3.7(2)31.358031, \\
 s^0(558) &= 558 = 2.3(2)31, \\
 s^1(558) &= 690 = s^1(462), \\
 s^0(560) &= 560 = 2(4)5.7, \\
 s^1(560) &= 928 = 2(5)29, \\
 s^2(560) &= 962 = 2.13.37.
 \end{aligned}$$

Additional information can be adduced: (a) $s^{44}(552) > 10^{10}$, (b) 558 is the next abundant number after 552 and occurs in no earlier sequence, (c) the sequence with leader 462 is the first in which the term 690 occurs, etc.

The reviewer has checked the table against his work with Selfridge [1]; no errors were found, but the following entry is missing (presumably a card was dropped).

132	1	204	2	1	1	2	3	17
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There are 40 instances of flag 9 (case (iii)): $n = 138, 276, 552, 564, 660, 702, 720, 840, 858, 936, 966, 1074, 1134, 1248, 1316, 1464, 1476, 1488, 1512, 1560, 1578, 1632, 1734, 1848, 1920, 1992, 2058, 2136, 2190, 2232, 2340, 2360, 2484, 2514, 2580, 2664, 2712, 2850, 2880, 2982$. The author has written (June, 1966) "As one might infer from [2], I started . . . to handle the 40 unfinished sequences . . . six were finished and a seventh joined one of the remaining. The remainder were pushed to . . . 4×10^{15} to 8×10^{20} ." His intention was to cover the range $n < 10^4$; this has now been done in [1]. It is known (D. H. and Emma Lehmer, written communication) that

$$s^{177}(138) = s^{300}(702) = s^{194}(720) = s^{167}(858) = s^{184}(936) = 1,$$

and ([1], and continuing calculations) that $s^{164}(1316) = s^{74}(2136) = s^{189}(2190) = s^{209}(3192) = s^{203}(4500) = s^{161}(5760) = s^{350}(5970) = s^{172}(6450) = s^{125}(8496) = s^{283}(8658) = s^{162}(9576) = s^{198}(9840) = 1$. These sequences all contain terms $> 10^{10}$.

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1. RICHARD K. GUY & J. L. SELFRIDGE, "Interim report on aliquot series," *Proc. Winnipeg Conf. on Numerical Math.*, October 1971.
2. G. AARON PAXSON, "Aliquot sequences" (preliminary report), *Amer. Math. Monthly*, v. 63, 1956, p. 614.

39[12].—B. A. GALLER & A. J. PERLIS, *A View of Programming Languages*, Addison-Wesley Publishing Co., Reading, Mass., 1970, vi + 282 pp., 24 cm. Price \$12.95.

Alan Perlis and Bernard Galler have been major contributors towards the design of programming languages. In this book, they present their views on the structure of programming, and describe a programming language, Algol D, that reflects these views. The book would be better entitled *A View of a Programming Language*, since they make little attempt to describe or deal with realistic programming languages other than Algol D. Thus, problems such as input-output and parallel processing are barely mentioned, and the reader may have a hard time seeing the connection between the ideas of this book and his favorite programming language. Nevertheless, the book must be regarded as a significant contribution to its field.

The expository quality is excellent. There are many exercises, and these are not only related to the text but are actually referenced within it. Thus, the reader who does the exercises will have extended the material covered by the book. The presentation is quite well-organized and ought to be suitable for a variety of readers, though some acquaintance with programming is surely necessary. Although the book is intended for classroom use as well as self-study, an instructor with well-formulated views on programming languages would probably not feel comfortable teaching from it because of its strong and rather personal biases.

There are four chapters. The first chapter is an elegant development of a higher-level (but not really practical) programming language starting with Markov algorithms. With the simplest type of algorithm as a base, the concepts of concatenation of algorithms, subroutines, operation on part of the data space, labelling, and storage addressing are introduced. The result of these successive extensions is the Addressed Labelled Markov Algorithm (ALMA). Emphasis is placed on the use of conventions in order to represent different data structures with the same set of characters. This chapter also introduces the concept of interpretation.

The second chapter is concerned with language. It begins with a discussion of flowcharting, and then moves on to present a version of Algol without arrays. Arrays are then presented in the context of a more general development of data structures, including strings as well. Unfortunately, the section on flowcharts contains a misleading explanation of an algorithm for finding a zero of a continuous function in an interval $[p, q]$. Although the description of the algorithm specifies that the interval must contain an odd number of zeros, the algorithm itself does not assume this. Under the stated assumption, the first half of the algorithm is superfluous.

The third chapter is concerned with data structures. In this chapter, three basic structure-forming operations are introduced, one for strings, one for arrays, and one for nonhomogeneous n -tuples. Since names (i.e., locations) of data are also allowed